## WHY MEASURES ARE MASS AND HOW MASS COUNTS

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## December 2016

## PART I: ASPECTS OF ICEBERG SEMANTICS

Iceberg semantics: meant as a useful framework for studying and developing theories of mass-count, singularity-plurality - for lexical nouns, complex nouns (NPs) and noun phrases (DPs).
Main inspirations:
-Link 1983, Landman 1991, and others: Boolean semantics for plurality.
-Chierchia 1998 (following in part Pelletier and Schubert): mass nouns with minimal elements
(furniture)- the supremum argument (the furniture $=$ the chairs and the tables).
-Rothstein 2010, Landman 2011: mass-count distinction based on overlap-disjointness.

- Krifka 1989: Count nouns based on natural units rather than atoms.
- Barbara Partee p.c. [public comments, many times]: Central idea of Boolean semantics:
not: singular noun denotes a set of atoms; but: singular noun denotes the set of minimal elements of the plural denotation.


## I.1. Boolean background

Boolean semantics: Link 1983:
-Boolean domains of mass objects and of singular and plural count objects. -Semantic plurality as closure under sum.

## Boolean interpretation domain B:

-Boolean algebra with operations of supremum $\sqcup$ (sum) and infimum $\sqcap$.

| Boolean part set: | $\mathbf{x} \boldsymbol{x} \boldsymbol{=}$ <br> $\mathbf{( X ]}=(\mathrm{b} \in \mathrm{B}: \mathrm{b} \subseteq \mathrm{x}\}$ | The set of all (Boolean) parts of x |
| :--- | :--- | :--- |
| Closure under $\mathrm{L}:$ | $* \mathrm{Z}=\{\mathrm{b} \in \mathrm{B}: \exists \mathrm{Y} \subseteq \mathrm{Z}: \mathrm{b}=\sqcup \mathrm{Y}\}$ | The set of all sums of elements of Z |
| Generation: X generates Z under $\sqcup$ iff $\mathrm{Z} \subseteq * \mathrm{X}$ and $\sqcup(\mathrm{X})=\sqcup(\mathrm{Z})$ <br> Every element of Z is a sum of elements of X and X and Z have the same <br> supremum |  |  |

Minimal elements: $\quad \min (X)=\{x \in X: \forall y \in X:$ if $y \sqsubseteq x$ then $y=x\}$
Atoms in B: $\quad$ ATOM $=\boldsymbol{m i n}(B-\{0\})$

| Disjointness: | $a$ and $b$ overlap $\mathbf{a}$ and $b$ are disjoint | iff $a \Pi b \neq 0 \quad$ a and $b$ have a part in common iff $\quad a$ and $b$ do not overlap |
| :---: | :---: | :---: |
|  | Z overlaps Z is disjoint | iff for some $a, b \in Z:$ a and $b$ overlap iff Z does not overlap |

## I.2. Mountain semantics

Mountain semantics: plural nouns are mountains rising up from singular nouns singular nouns are sets of atoms (the bottom of the structure)
Since Ronya has no proper parts in the count domain, her mass parts are in a different domain ordered by a different part-of relation: sorting.

-counting in terms of atoms: x is three cats $=\mathrm{x}$ has three atomic cat parts
-distribution in terms of atoms: each of the cats $=$ each of the atomic cat parts

## Correctness of counting atoms:

If A is a set of atoms then *A has the structure of a complete atomic Boolean algebra with A as set of atoms. This allows correct counting.

Consequence of sorted domains (Landman 1989, 1991):

1. Basically no relation between $\sqsubseteq$ and intuitive lexical part-of relations:

Ronya, Ronya's front leg, Ronya's paw
The stuff making up Ronya is not part of Ronya Ronya is an atom
2. The problem of portions: portions are countable mass
(1) a. The coffee in the pot and the coffee in the cup were each spiked with strychnine.
b. I drank two cups of coffee

I didn't ingest the cups, so I drank two portions of coffee
Problem: coffee is uncountable stuff, each portion of coffee is coffee mass + mass $=$ mass, so how can you count portions of coffee?
Landman 1991: portion shift shifts mass stuff to count atoms.
Iceberg semantics: different view on mass-count, not relying on atoms.

### 1.3. Iceberg semantics

1. Iceberg semantics stays as close to Mountain semantics as possible.

Nouns are interpreted as icebergs: their interpretation consist of a body and a base.
-body = the interpretation in Mountain semantics.
-base $=$ set that generates the body under sum: the basic stuff that body objects are made of.
Count icebergs: cats: body: closure under sum of the base
base: set of singular cats
$=$ The set of objects in terms of which cat-pluralities are counted.
Iceberg semantics: Plural body is a mountain rising up from the singular base.
The base is not a set of atoms but floats in a sea of mass: an iceberg.
2. No sorting: the same body is mass or count depending on the base it is grounded in. the same body is singular or plural depending on the base it is grounded in.
$\rightarrow$ 3. mass - count: disjointness of the base instead of atomicity.
$\rightarrow 4$. Compositional semantic: notions mass and count also apply to complex NPs and DPs.
NPs are interpreted as iceberg sets [i-sets]:
An i-set is a pair of sets $X=\langle\operatorname{body}(X), \operatorname{base}(X)>$ with the body generated by the base under sum
Correctness of counting is not to do with atomicity itself but with disjointness:

## Correctness of counting:

If Z is disjoint then $* \mathrm{Z}$ has the structure of a complete atomic Boolean algebra with Z as set of atoms. This allows correct counting.
the cats:
count: sum of disjoint set $\mathrm{CAT}_{\mathrm{wt}}$
mass: sum of minimal identifiable cat-stuff


No sorting: -mass entities and count entities stand in the same part-of relation -sets of 'mass' portions can be count if the grammar makes them disjoint.

### 1.4. The mass-count distinction

## COUNT

Counts NPs are interpreted as count i-sets.
i-set $X=\langle\operatorname{body}(X)$, base $(X)>$ is count iff base $(X)$ is disjoint, otherwise mass.

Plural count NP cats:
cats $\rightarrow$ CATS $_{\mathrm{wt}}=\left\langle * \mathrm{CAT}_{\mathrm{wt}, \mathrm{CAT}_{\mathrm{wt}}>}>\right.$
$\mathrm{CAT}_{\mathrm{wt}}=\operatorname{base}\left(\right.$ CATS $\left._{\mathrm{wt}}\right)$
Grammatical requirement: $\mathbf{C A T}_{\mathbf{w t}}$ is a disjoint set.
In context, we choose $\mathrm{CAT}_{\mathrm{wt}}=\{$ ronya, shunra, emma, pim $\}$ disjoint:


We get the same Boolean structure as in Mountain semantics, but based on a disjoint set.

## NEAT MASS

Neat mass NPs are (in normal contexts) interpreted as neat mass i-sets.
i-set $X=\langle\boldsymbol{b o d y}(X), \boldsymbol{b a s e}(X)>$ is neat $\operatorname{iff} \boldsymbol{\operatorname { m i n }}(\mathbf{b a s e}(X))$ is disjoint and generates base $(X)$ under sum; otherwise mess.

## Neat mass NP kitchenware:

kitchenware $\rightarrow$ KITCHENWARE $E_{\mathrm{wt}}=\left\langle * \mathrm{KW}_{\mathrm{wt}}, \mathrm{KW}_{\mathrm{w}, \mathrm{t}}\right\rangle$
$\mathrm{KW}_{\mathrm{w}, \mathrm{t}}=\operatorname{base}\left(\right.$ KITCHENWARE $\left._{\mathrm{wt}}\right)$
Grammatical requirement: neat: $\boldsymbol{\operatorname { m i n }}\left(\mathrm{KW}_{\mathrm{wt}}\right)$ is disjoint mass: $\mathrm{KW}_{\mathrm{wt}}$ is not disjoint
$\min \left(\mathrm{KW}_{\mathrm{wt}}\right)=\{$ teapot, cup, saucer, pan $\}$ is disjoint
$\mathrm{KW}_{\mathrm{wt}}=\{$ teapot, cup, saucer, cup and saucer, teaset, pan $\}$ is not disjoint. "Items sold as one"


The same structure as for count, but counting is not felicitous because the base is not disjoint.

## MESS MASS

Mess mass NPs are (in normal contexts) interpreted as i-sets that are neither count nor neat mass:
i-set $X=<\operatorname{body}(X)$, base $(X)>$ is mess mass iff base $(X)$ is not disjoint and not generated by a disjoint set of minimal elements (for instance, because there are no minimal elements.)

$$
\begin{array}{r}
\text { water } \rightarrow W A T E R_{\mathrm{wt}}=\left\langle\mathrm{WATER}_{\mathrm{wt}}, \text { base }\left(W A T E R_{\mathrm{wt}}\right)\right\rangle \\
\text { with } W A T E R \subseteq \boldsymbol{b}^{\operatorname{base}\left(W A T E R_{\mathrm{wt}}\right)}
\end{array}
$$



WATER $_{\mathrm{wt}} \neq$ Set of water molecules

$$
=\text { Water molecules }+ \text { space around them and inside them }
$$

$\operatorname{Body}\left(W A T E R_{\mathrm{wt}}\right)=$ set of regions of space with water molecules distributed in it.
Example choice of base:
base $\left(W_{A T E R}^{w t}\right)=$ Set of sub regions that contain one water molecule.
Intuition: if you region contains only half a water molecule, or an H atom, it doesn't count as water.

Fact 1: base( $\mathrm{WATER}_{\mathrm{wt}}$ ) is not disjoint (many sub regions contain the same molecule).
Fact 2: base( $\mathrm{WATER}_{\mathrm{wt}}$ ) generates $\mathrm{WATER}_{\mathrm{wt}}$ under sum.
 away some space from a region containing one molecule).

Hence $W A T E R_{\mathrm{wt}}$ is mess mass.

## I.5. Disjointness and counting.

Lexical semantics of numericals and sorted count quantifiers makes reference to distribution set $\mathbf{D}$ which presupposes disjointness:

Presuppositional distribution: $\mathrm{D}_{\mathrm{Z}}(\mathrm{x})$
$\mathbf{D}=\lambda Z \lambda \mathrm{x} . \begin{cases}\mathbf{x} \mathbf{x} \cap \mathrm{Z} & \text { if } \mathrm{Z} \text { is disjoint } \\ \perp & \text { otherwise }\end{cases}$
$D_{Z}(x)$ is the set of Z-parts of $x$, presupposing that $Z$ is disjoint
Counting as presuppositional cardinality:
$\operatorname{card}=\lambda Z \lambda \mathrm{x} .\left|\mathrm{D}_{\mathrm{Z}}(\mathrm{x})\right|$
$\operatorname{card}_{Z}(x)$ : the cardinality of the set of $Z$-parts of $x$, presupposing that $Z$ is disjoint.
Consequences for count versus mass:

1. Counting: $\quad \checkmark$ three cats $\quad$ \#three mud
2. Distribution: $\quad \checkmark$ each of the cats $\quad$ \#each of the mud
3. Comparison: most cats purr: only cardinality comparison most mud is clay: only measure comparison
But see Part Three!

### 1.6 Compositional semantics of bases:

Head principle for NPs: COMPLEX NP = Interpretation of a complex NP
$H E A D=\quad$ Interpretation of its head
$\operatorname{base}($ COMPLEX NP $)=(\operatorname{body}($ COMPLEX NP $) \mathbf{]} \cap \operatorname{base}(H E A D)$
the base of the complex $=$ the set of all Boolean parts of body $(C O M P L E X N P)$ intersected with the base of the head

Head Principle for NPs: Base information is passed up from the head NP to the complex NP
Consequences of the head principle for mass count:
Fact: If base (HEAD) is disjoint, then base(COMPLEX NP ) is disjoint (intersection)
Corollary: Mass-count: The mass-count characteristics of the head inherit up to the complex:
Complex noun phrases are count if the head is count.
Complex noun phrases are mass if the head is mass.
SOME COMPOSITIONAL DERIVATIONS: ON THE POWERPOINT

## I.7. Count interpretations of complex nouns phrases: classifiers

Classifier structure:


Classifier interpretation:
INTERSECT
I


## 1. Container classifier interpretation:

(2) a. There was also the historic moment when I accidentally flushed a bottle of lotion down the toilet. That one took a plumber a few hours of manhandling every pipe in the house to fix. $[\gamma]$

Noun bottle shifts to a container classifier: function from sets to sets container[bottle]: function mapping stuff Z onto bottles containing Z

Head: container $[$ bottle $](Z)$ is a disjoint set of bottles containing $Z$

Three bottles of wine
Interpretation: body: three bottle-containers each containing wine
base: set of disjoint bottle-containers
Fact: Disjoint base: The container classifier interpretation of noun phrase bottle of wine is count.

## 2. Contents classifier interpretation:

(2) b. I drank three glasses of beer, a flute, a pint, and a stein.

Noun bottle shifts to a contents classifier: function from sets to sets
contents[bottle]: function mapping stuff $Z$ onto portions of $Z$ that are contents of bottles
Head: contents $[$ bottle $](\mathbf{Z})$ is a set of portions of $Z$ that are contents of bottles.
This is a disjoint set, since disjoint bottles have disjoint contents.

## Three bottles of wine

Interpretation: body: three portions of wine each of which is the contents of a bottle -container
base: set of disjoint portions which are the contents of bottle-containers
Fact: Disjoint base: The contents classifier interpretation of noun phrase bottle of wine is count.
So: bottle of wine denotes wine, 'mass' stuff, but is count, it denotes a set of disjoint portions. More discussion of classifier and portion interpretations, see Landman 2016a.

## PART II: WHY MEASURES ARE MASS

## II.1. Measure interpretations are mass [Rothstein 2011, Landman 2016a]

Background: Partitives with singular DPs patterns with partitives with mass DPs:
(3) a. $\checkmark$ much/\#each of the wine
cf. $\checkmark$ each of the cats
b. $\checkmark$ much/\#one of the cat
cf. $\checkmark$ one of the cats $/ \checkmark$ all ten of the cats
Suggestion: If we assume that the semantics of partitives disallows singular i-objects, then partitives with singular DPs can become felicitous only by shifting the singular object to a mass object (by changing the base): opening up internal structure:
(4) After the kindergarten party, much of my daughter was covered with paint. (shift opening up the surface area of my daughter + much - area measure)

This shift is obligatory for partitives with singular DPs. Plural cases can be found:
(5) While our current sensibilities are accustomed to the tans, taupes, grays and browns, in their time much of the rooms as well as the cathedral proper would have been beautifully painted. [ $\gamma$ ]
[ $\gamma$ ] means: googled
But plural cases are rare, and not everybody (e.g. Susan Rothstein) accepts cases like (5).
Crucial here: sharp contrasts between plural opening up (6b) and measure phrases (6c):
(6) a. \#Much ball bearings was sold this month.
b. \#?Much of the ball bearings was sold this month.
c. $\checkmark$ Much of the ten kilos of ball bearings was sold this month.

So: the felicity of (6c) is not to do with opening up (as in (6b), but with the measure phrase. Cf. also (7) (based on examples from Rothstein 2011):
(7) a. Many of the twenty kilos of potatoes that we sampled at the food show were prepared in special ways.

20 one kilo-size portions - count
b. Much of the three kilos of potatoes that I ate had an interesting taste.
potatoes to the amount of 3 kilos - mass
Rothstein 2011: Partitive NPs with measure phrases pattern with mass nouns.

## II.2. The body of the measure and the body of the measure phrase

 [Landman 2016a]Classifier structure: mismatched with: Measure interpretation:


body of the measure phrase: interpretation with function composition:
(numerical $\circ$ measure) $\cap$ complement.
three liter wine
three composes with liter, the result intersects with wine
base of the measure phrase: head principle: base $(C)=(\operatorname{body}(C)] \cap \operatorname{base}(H)$
(numerical $\circ$ measure) $\cap$ complement.
$\left(\lambda \mathrm{n} . \mathrm{n}=3 \quad \circ \operatorname{liter}_{\mathrm{wt}}\right) \cap$ WINE $_{\mathrm{wt}} \quad=$
three liters of wine $\rightarrow$ <body, base>

$$
\text { body }=\lambda x . \operatorname{liter}_{w t}(x)=3 \wedge \operatorname{WINE}_{\mathrm{wt}}(\mathrm{x})
$$

Wine to the amount of three liters
entities that are wine and measure three liters

## II.3. The base of the measure.

Measure functions: functions from B into $\mathrm{R}^{+}$, the set non-negative real numbers, setting 0 to 0 :
$\boldsymbol{\mu}_{\mathrm{wt}}: \mathrm{B} \rightarrow \mathbf{R}^{+} \cup\{\perp\}$ where $\boldsymbol{\mu}_{\mathrm{wt}}(0)=0$

Measures denote additive continuous measure functions (liter, meter, broadloom meter, ...)
(broadloom meter measures the length of carpet with fixed width 3.66 meter. Interesting measure, because it has many parts for which the measure is undefined)

Additivity: I assume a standard definition which entails Boolean addition:

## Boolean addition:

The measure value of $x \sqcup y$ is the arithmetic sum of the measure values of $x-y, y-x$ and $x \sqcap y$
Continuity: I assume a standard definition of continuity for measure functions which entails the standard intermediate value theorem: (I will use the theorem).

## Intermediate Value Theorem:

When a body grows from x with measure $\boldsymbol{\mu}_{\mathrm{wt}}(\mathrm{x})$ to y with measure $\boldsymbol{\mu}_{\mathrm{wt}}(\mathrm{y})$, then between x and y the measure passes through all intermediate measure values:
each r with $\boldsymbol{\mu}_{\mathrm{wt}}(\mathrm{x})<\mathrm{r}<\boldsymbol{\mu}_{\mathrm{wt}}(\mathrm{y})$ is the measure value of some part x with $\mathrm{x} \sqsubseteq \mathrm{z} \sqsubseteq \mathrm{y}$

Fitting measures into Iceberg semantics: A function is a set of ordered pairs:
Measure function $\boldsymbol{\mu}_{\mathrm{wt}}$ is a set of object-measure value pairs in $\mathrm{B} \times\left(\mathbf{R}^{+} \cup\{\perp\}\right)$

Proposal: Generalize the notion i-set to measure i-set:
Measure i-sets: Given measure function $\boldsymbol{\mu}_{\mathrm{wt}}$.
A measure $i$-set is a pair $X=\langle\boldsymbol{\operatorname { b o d }} \mathbf{y}(X)$, $\operatorname{base}(X)>$, where $\operatorname{body}(X)$ and base $(X)$ are sets of objectmeasure value pairs, and base $(X)$ generates body $(X)$ under sum.

Requires lifting the Boolean structure of $\mathbf{B}$ to the set of object-measure value pairs (trivial):
$\mathrm{B} \boldsymbol{\mu}_{\mathrm{wt}}=\left\{\left\langle\mathrm{b}, \boldsymbol{\mu}_{\mathrm{wt}}(\mathrm{b})\right\rangle: \mathrm{b} \in \mathrm{B}\right\}$
$\left.\left\langle\mathrm{x}, \boldsymbol{\mu}_{\mathrm{wt}}(\mathrm{x})\right\rangle \sqsubseteq\left[\mathrm{B} \boldsymbol{\mu}_{\mathrm{wt}}\right]<\mathrm{y}, \boldsymbol{\mu}_{\mathrm{wt}}(\mathrm{y})\right\rangle$ iff $\mathrm{x} \sqsubseteq_{\mathrm{B}} \mathrm{y}$
$\left\langle\mathrm{x}, \boldsymbol{\mu}_{\mathrm{wt}}(\mathrm{x})\right\rangle \sqcup\left[\mathrm{B} \boldsymbol{\mu}_{\mathrm{wt}}\right]\left\langle\mathrm{y}, \boldsymbol{\mu}_{\mathrm{wt}}(\mathrm{y})\right\rangle=\left\langle\mathrm{x} \sqcup_{\mathrm{B}} \mathrm{y}, \boldsymbol{\mu}_{\mathrm{wt}}(\mathrm{x} \sqcup \mathrm{y})\right\rangle$
Proposal: Interpret measure liter as a measure i-set with body the additive continuous volume measure function liter ${ }_{\mathrm{wt}}$ and find a generating base.
$\square$
[measure liter $] \rightarrow$ LITER ${ }_{\mathrm{wt}}=\left\langle\operatorname{body}\left(\right.\right.$ LITER $\left._{\mathrm{wt}}\right)$, base $\left(\right.$ LITER $\left.\left._{\mathrm{wt}}\right)\right\rangle$ with:
1.body $\left(L I T E R_{\mathrm{wt}}\right)=\operatorname{liter}_{\mathrm{wt}}$
2. base(liter ${ }_{w t}$ ) is a subset of liter ${ }_{w t}$ that generates liter ${ }_{\mathrm{wt}}$ under sum

## Main Result:

If $\left\langle\boldsymbol{\mu}_{\mathrm{wt}}, \mathbf{D B}\right\rangle$ is a measure i-set
where $\boldsymbol{\mu}_{\mathrm{wt}}$ is an additive continuous measure function and DB is a disjoint subset of $\boldsymbol{\mu}_{\mathrm{wt}}$ then DB contains only pairs of the form $\langle\mathbf{x}, \mathbf{0}\rangle$ or $\langle x, \perp\rangle$.

Proof: This follows from the Intermediate Value Theorem.
Intuition: If $\langle x, r\rangle \in \mathbf{D B}$ then proper parts of x with lower values exist and must be generated by the base under sum. This forces the base to overlap.

This almost proves that the base of the measure is not disjoint and hence that measures are (mess) mass. But not quite by itself.
-The theory does not disallow 'infinitesimal point objects':
Think of models for space and time (e.g. Tarski's algebra of solids for Euclidian geometry).
We can represent time intervals and space solids as infinite sets of point: regular open sets of points. If we include the points in the model they don't have positive volume values. -So we could generalize this to matter and generate all measure values from a disjoint set of points just with ப.

But note: these would not be points of time, space, space-time, they would be points of matter: a bit like the atoms of Demokritos.

## Motivation of iceberg semantics:

Try to develop the semantics of mass nouns and count nouns in naturalistic structures.
Try not to disregard natural parts and structure. Try not to include non-natural structure.
-Example of less parts than is reasonable: Lønning 1987 Homogeneity:
In Lønning's structures: liquid only has parts that are liquid
yellow only has parts that are yellow
yellow liquid only has parts that are yellow liquid,
even if yellow is a property that stuff only has in a certain bulk.
Diagnosis: Natural parts are ignored for the sake of Lønning's definition of homogeneity.
-Example of more parts than is reasonable: Bunt, ter Meulen, Landman 1991 (and many others).
Divisibility: semantically water can be partitioned ad infinitum into parts that are themselves water.
Landman 2011:
(8) There is salt on the objective of the microscope, [one molecule worth] mass noun salt

Divisibility requires that the denotation of mass noun salt also in (9) divides into parts that are salt: it's salt all the way down. But nature doesn't have such parts (Homeopathic semantics).

Dogma of Iceberg Semantics: points of matter are exactly the kind of non-naturalistic objects we want to do without Iceberg semantics rejects points of matter.

Corollary: Continuous additive measures are interpreted as mess mass measure i-sets: measure i-sets with an overlapping base.

In other words:
Measures are interpreted as mess mass i-sets

## II.4. The base of the measure, a suggestion.

What is base $\left(\right.$ LITER $\left._{\mathrm{wt}}\right)$ ?
Intuitively: the base contains the 'contextually minimal relevant ' stuff that the body is made of.
Above discussion: for measure functions, the generating base is closed under parts.
Since measures are extensional (they don't distinguish objects of the same size), , we think of the base as the set of all part-measure value pairs whose measure value is smaller than a certain value.


Let $\mathbf{m}$ (short for $\mathbf{m}_{\text {liter,wt }}$ ) be a contextually given measure value. For concreteness think of $\mathbf{m}$ as the lowest volume that our experimental precision weighing scales can measure directly (rather than extrapolate).
liter-up-to- $\mathbf{m}_{w t}=\left\{\left\langle\mathrm{x}, \operatorname{liter}_{\mathrm{wt}}(\mathrm{x})\right\rangle\right.$ : $\left.\operatorname{liter}_{\mathrm{wt}}(\mathrm{x}) \leq \mathbf{m}\right\}$
The set of object-liter value pairs where the liter value is less than or equal to $\mathbf{m}$.
We set:
$[$ measure liter $] \rightarrow\left\langle\right.$ liter $_{\mathrm{wt}}$, liter-up-to- $\left.\mathrm{m}_{\mathrm{wt}}\right\rangle$
Fact 1: liter-up-to- $\mathbf{m}_{\mathrm{wt}}$ overlaps (since it is closed downwards)
Fact 2: liter-up-to- $\mathbf{m}_{\mathrm{wt}}$ contains no minimal elements (continuity)
Fact 3: liter-up-to- $\mathbf{m}_{w t}$ generates liter $_{\mathrm{wt}}$ under sum ( $\sqcup$ is the complete supremum operation)

## II.5. The base of the measure phrase.

In the derivation we keep track in the base of the measure function (measure i-set base), but lower the body to an i-set body (with lowering operation ${ }^{\downarrow}$, details in Landman 2016b). We derive:

## three liters of wine :

$$
\text { body }=\lambda \mathrm{x} \cdot \operatorname{liter}_{\mathrm{wt}}(\mathrm{x})=3 \wedge \mathrm{WINE}_{\mathrm{wt}}(\mathrm{x})
$$

Wine to the amount of three liters
${ }^{\downarrow}$ base $=\lambda y . \mathrm{y} \subseteq \sqcup\left(\lambda \mathrm{x}\right.$. $\left.\mathrm{WINE}_{\mathrm{wt}}(\mathrm{x}) \wedge \operatorname{liter}_{\mathrm{wt}}(\mathrm{x})=3\right) \wedge \operatorname{liter}_{\mathrm{wt}}(\mathrm{y}) \leq \mathbf{m}$
Stuff that is part of the wine and has volume at most $m$

Fact: three liters of wine on the measure interpretation is mess mass.

## Reason:

-The head of the construction is not wine but liter.
-The base of liter is the set of all entities measuring at most $\mathbf{m}$.
-When we intersect the set of all Boolean parts of the sum of the wine with the base of liter we get the set of all Boolean parts of the wine that measure less than $\mathbf{m}$.
-This set is not disjoint.
Hence, we derive Rothstein's observation:
Measure interpretations are mess mass interpretations.

Note:

## 500 grams of bonbons:

body: Set of sums of bonbons that weigh 500 grams.
${ }^{\downarrow}$ base: Set of Boolean parts of the sum of bonbons that weigh less than $m$ grams.
500 grams of bonbons is mass relative to the measure base.
The body is not ground into mass, it stays a sum of singular bonbons.
The measure base makes the measure phrase mass.
(9) [at Neuhaus in the Galerie de la Reine in Brussels]

Customer: Ik wou graag 500 gram bonbons. Shop assistant: Eén meer or één minder? I would like 500 grams of pralines. One more or one less?

- Ah, just squeeze enough into the box so that it weights exactly 500 grams.

$$
\left(\circlearrowleft^{*}=\text { faux pas }\right)
$$

## PART III: WHEN MASS COUNTS

$\underset{\times}{\otimes}$. Caveat: Despite appearances, no animals were harmed in the research for this section. ${ }_{\times 1}^{*}$

## III.1. Counting mess mass

Count expressions that make reference to $\left.\mathbf{D}_{\text {base }(\text { HEAD })}\right)(\mathrm{x})$ : the set of base $($ HEAD $)$ parts of $\mathbf{x}$ :
Counting and disjointness:
numerical three involves: $\quad \lambda \mathrm{x} . \operatorname{card}_{\text {base }(H E A D)}(\mathrm{x})=3 \quad \mathbf{D}_{\text {base }(H E A D)}$
Distribution and disjointness:
Distributor each involves:
$\lambda \mathrm{x} . \forall \mathrm{a} \in \mathbf{D}_{\text {base }(H)}(\mathrm{x}): \varphi(\mathrm{a}) \quad \mathbf{D}_{\text {base }(H E A D)}$
Comparison reading for count most:

| (10) Most farm animals are outside in summer. | D $_{\text {base }(H E A D)}$ |
| :--- | :--- |
| $\operatorname{card}_{\text {base }(H E A D)}(\sqcup(\lambda \mathrm{x}$. body $(H E A D)(\mathrm{x}) \wedge \varphi(\mathrm{x})))>$ |  |
| $\operatorname{card}_{\text {base }(H E A D)}(\sqcup \mathbf{b o d y}(H E A D)-\sqcup(\lambda \mathrm{x} . \operatorname{body}(H E A D)(\mathrm{x}) \wedge \varphi(\mathrm{x})))$ |  |

Presupposition: base $(H E A D)$ is disjoint. hence $H E A D$ is count.
Puzzle: distribution and count comparison are not restricted to count nouns:

1. Stubbornly distributive adjectives (Rothstein 2011, Schwarzschild 2009).

Schwarzschild: big strongly disfavor collective interpretations, as compared to noisy. Rothstein: neat mass noun furniture combines with big, and big is distributive (like each):
(11) a. The noisy boys $=\checkmark$ the boys that are noisy $-\quad \checkmark$ the noisy group of boys
b. The big chairs $=\checkmark$ the chairs that are big $-\times$ the big group of chairs
c. The big furniture $=\checkmark$ the pieces of furniture that are big
$\times$ the big group of furniture pieces
$=$ distributivity for neat mass nouns
2. Cardinal comparison: Barnes and Snedeker 2005: speakers readily get cardinality comparison for neat mass nouns. (but note: mass measure interpretations are also possible).
(12) a. Most farm animals are outside in summer. [Landman 2011]
b. Most livestock is outside in summer.
(12a) only has a count comparison reading.
(12b) allows comparison, say, in terms of volume or size of biomass, i.e. a measure comparison that is normal for mess mass nouns. But also a prominent cardinality comparison reading.
= cardinal comparison with most for neat mass nouns.

## This section:

-In Dutch, in context, stubbornly distributive adjectives can modify mess mass nouns
-In Dutch, in context, cardinal comparison with most is possible for mess mass nouns -The contexts are contexts where disjoint portioning is strongly contextually salient.

Examples do occur in English, but are admittedly hard to find:
(13) It's not that I can't cook, but I lack experience with preparing big meat and elaborate meals. [ $\gamma]$

Dutch: Even though groot/big patterns with English on the data in (12) above, searching the web, convincingly shows that the Dutch go with Slagerij Franssen:
(14) Slagerij Franssen, Maastricht: Tips voor het bereiden van groot vlees.

Het bereiden van groot vlees lijkt voor velen een groot probleem. Liever kiest men dan voor een biefstukje of een filet. Echter, groot vlees heeft veel voordelen! $[\gamma]$

Butcher shop Franssen, Maastricht: Tips for preparing big meat.
Many seem to regard preparing big meat as a big problem. And so they tend to choose a steak or a filet instead. However, big meat has many advantages!

Vlees in Dutch is a mess mass noun, like meat in English.
(15a) shows that groot/big is compatible with mess mass nouns like vlees/meat in Dutch and has a distributive interpretation. But: no shift to a count noun is involved, as shown in (15b-c):
[difference with count shifted mass nouns as in drie bier/three beer - drie patat/three french fry]
(15) a. Het grote vlees ligt in de linker vitrine, het kleine vlees in de rechter vitrine.

The big meat lies in the left display compartment, the small meat in the right one.
b. \#Drie groot vlees \#Drie grote vlezen \#Three big meat \#three big meats
c. $\checkmark$ Het meeste van het grote vlees $\checkmark$ is kameel/ \#De meeste van het grote vlees zijn kameel. $\checkmark$ Most $_{\text {[mass] }}$ of the big meat is camel \#Most $_{\text {[count] }}$ of the big meat are camel

We look at cardinality comparison with mess mass nouns like rijst/rice or vlees/meat:
Out of the blue, Dutch does not allow count comparison (like English):
(16) De meeste rijst is bruin.


Most rice is brown
not so many very large grains of white rice
very many very small grains of brown rice

Out of the blue: (16) is false.
Out of the blue: mass comparison in terns of volume, not count cardinality comparison.

But if we set up the context carefully we can trigger count readings.
Example adapted from an example by Peter Sutton p.c.:
We are playing a game in which we hide small grains of brown rice and very large grains of white rice (to make it not too difficult for the children). Winner is the one who finds the largest number of grains of rice. The numbers and sizes are as in the above picture. Now, as it turns out, Peter is very good at this game. In fact after the game, we take stock and declare:
(17) De meeste rijst is in het bezit van Peter.

Most rice is in the possession of Peter.
In this context: (17) is true and felicitous, even if Peter only found small grains. This interpretation involves count comparison.

Rationale: The context has made the grid grain available:
-Count comparison in terms of the cardinality of elements in the grid.
-Grids are partitionings into disjoint portions.
-Count comparison via portions is possible in Dutch for mess mass nouns when the portioning is made salient in context.

We show the same with vlees-meat: Below is the display compartments of our butcher shop:

Left compartment: hunks of veal

(18) Het meeste vlees ligt in de rechter vitrine.

Right compartment: hunks of baby duck.


Most meat lies in the right display compartment.

Out of the blue: (18) is false.
Out of the blue: (18) requires mass comparison in terms of volume:
Count comparison is not natural at all.

We add context:

Context: Tonight you celebrate your Traditional Family Dinner, at which the two Parents eat the Traditional Meal of veal and the twelve Children eat, by Tradition, baby duck. Hence, you have ordered what is in the above display compartments (which is in fact all the veal and duck we have left in the shop).

Disaster strikes the butcher shop: the hunks of baby duck were found out to be infected with worms. They have to be destroyed, and can't be sold. I call you with the following message:
(19) Er is een probleem met uw bestelling. Het meeste vlees bleek besmet te zijn met wormen. We moesten het wegdoen, en we hebben geen tijd om vandaag nog een nieuwe bestelling binnen te krijgen.

There is a problem with your order. Most (of the) meat turned out to be infected with worms. We had to get rid of it. and we don't have time to get a new order in by today.

In this context: (19) is felicitous and true.
In this context: reading for the mess mass noun that involves count comparison in terms of contextual portions, the hunks of meat in the display compartments.
Count comparison is possible.

One more case: we compare groot vlees/big meat in the compartments

## Left compartment:

Small hunks of baby duck
Big hunks of pork


## Right compartment: Exotic meat

Small hunks of baby penguin
Huge hunks of elephant steak

(20), out of the blue, with contrastive stress on groot/big:
(20) Het meeste grote vlees ligt in de linker vitrine.

Most big meat lies in the left display compartment.
Out of the blue: (20) is felicitous and true without extra context:
Count comparison of big hunks of meat is possible.
We observe:
(18): out of the blue only a mess mass reading.
(19): counting reading by creating a context that made counting portions salient.
(20): we don't need to set up that special counting context.

## Explanation:

-Count comparison with mess mass nouns requires portion shift, shift to salient portions that can be counted.
Portion counting context is required to make this shift salient.
-Semantics of groot/big involves distribution, which itself requires a salient disjoint distribution set to be made available. Mess mass nouns: such a disjoint set is only available via portion shift.

But then: The semantics of groot vlees/big meat already involves portion shift.
No further context needed for counting comparison in (20).

## III.2. How mass counts

We show why (in Iceberg semantics) distributivity is possible in the mass domain and propose an analysis of how it works there. (Extension to count comparison is straightforward.)

> groot/big is distributive, can modify mass nouns, and does not shift the mass noun it modifies into a count noun:
> groot meubilair/big furniture and groot vlees/big meat are mass NPs.

Iceberg semantics: the mass nature of the interpretation of groot vlees/big meat follows from the Head principle:
vlees $\quad \rightarrow<\operatorname{MEAT}_{\mathrm{wt}}, \operatorname{base}\left(M E A T_{\mathrm{wt}}\right)>$, with mess mass base $\left(M E A T_{\mathrm{wt}}\right)$.
groot vlees $\rightarrow$ body: meat that comes in portions each of which is big base: the part set of the sum of that body intersected with the mess mass base.
i.e. the base is the set of all parts of the meat making up the big portions that are in base $\left(M E A T_{\mathrm{w}, \mathrm{t}}\right)$. This is an overlapping base.

Hence: the interpretation of groot vlees is mess mass.

Counting, distribution, count comparison for count nouns:
Restriction to count predicates: the semantics involves $\mathbf{D}_{\text {base }(H E A D)}$ or $\operatorname{card}_{\text {base }(H E A D)}$, which presupposes that base $(H E A D)$ is disjoint.
Hence $H E A D$ is required to be count.

## Crucial observation:

The operators defined in Iceberg semantics are $\mathbf{D}_{\mathbf{z}}$ and $\operatorname{card}_{\mathbf{z}}$, where $\mathbf{Z}$ is a disjoint set.
The operations are not themselves linked to base(HEAD).
Hence: The Iceberg semantics involving $\mathbf{D}$ and card must provide a disjoint set But this doesn't have to be base( $H E A D$ ).

## The big picture:

The semantics of modifier big is based on set big $\boldsymbol{H E A D}^{\text {, the general form of which is: }}$

$$
\begin{aligned}
\operatorname{big}_{\text {HEAD }}= & \lambda \mathrm{x} . \operatorname{body}(\boldsymbol{H E A D})(\mathrm{x}) \wedge \forall \mathrm{a} \in \mathbf{D}_{\mathbf{z}}(\mathrm{x}): \mathrm{BIG}_{\mathrm{wt}}(\mathrm{a}) \text { presupposition: } \mathbf{Z} \text { is disjoint } \\
& \text { The set of body-HEAD entities all of whose Z-parts are big }
\end{aligned}
$$

Semantics of count nouns in English and Dutch: Identification of $\mathbf{Z}$ with base(HEAD):

Count: $\mathbf{Z}=\operatorname{base}($ HEAD $)$
$\operatorname{big}_{\text {HEAD }}=\lambda \mathrm{x} . \operatorname{body}(\boldsymbol{H E A D})(\mathrm{x}) \wedge \forall \mathrm{a} \in \mathrm{D}_{\text {base }(H E A D)}(\mathrm{x}): \mathrm{BIG}_{\mathrm{wt}}(\mathrm{a})$
presupposition: base $(H E A D)$ is disjoint
The set of body-HEAD objects all of whose base(HEAD)-parts are big

Mass nouns: Identification of $\mathbf{Z}$ with base (HEAD) is impossible, since base $(\boldsymbol{H E A D})$ is not disjoint. This means: for big to felicitously modify a mass noun, another interpretation for $\mathbf{Z}$ must be found.

Neat mass nouns: kitchenware or livestock (Landman 2011):
-Base is not disjoint, but it is in general not difficult to find a salient disjoint subset of the base (or modify the base and make its elements in context disjoint, see Landman 2017ms).
-Neat mass nouns: One subset that is always available is, (by definition of neat mass) the disjoint set of minimal elements of the base: $\boldsymbol{\operatorname { m i n }}($ base $(H E A D)$ ).

Landman 2011: kitchenware and furniture: [contextually itemized]
$\mathbf{Z}$ can be linked to different salient disjoint subsets of the base.
Landman 2017ms: livestock, poultry, animate neat mass nouns
$\mathbf{Z}$ is always linked to $\mathbf{m i n}(\mathbf{b a s e}(H E A D))$.
Het grote veelthe big livestock is the sum of big sized farm animals.
Landman 2017ms: the same is true for count comparison:
Count comparison of kitchenware is context dependent
Count comparison of vee/livestock count compares the cardinality of sets of farm animals, i.e. subsets of $\min ($ base $(H))$ :
(21) a. Het meeste vee is 's zomers buiten.
b. Most livestock is outside in summer.

Mess mass nouns: groot vlees/big meat.
No salient disjoint set available, not even $\min (b \operatorname{base}(H))$.
The only way to find a disjoint set is through contextual portioning.
Dutch: If, in context, PORTION $_{c}$ makes a disjoint set PORTION ${ }_{c w t}$ salient, then the semantics allows $\mathbf{Z}$ in $\mathbf{D}_{\mathbf{z}}$ to pick up:
$\lambda \mathrm{x}$.PORTION $\mathrm{cmt}^{\mathrm{cwt}}(\mathrm{x}) \wedge \operatorname{body}(P)(\mathrm{x}) \quad$ the disjoint set of portions of body-P in PORTION ${ }_{\mathrm{cwt}}$
We derive a mess mass interpretations of groot vlees/big meat
meat that comes in the form of big portions, generated by its mess mass meat-base.
Similar for the choice of $\mathbf{Z}$ in $\operatorname{card}_{\mathbf{Z}}$ in counting comparison interpretations of most.

Note 1: Not explained: Why is this easy for Dutch mess mass nouns and hard in English. Only explained: what happens, if and when it happens.

Note 2: The fact that English numericals like at least three and English distributor each cannot apply to mass nouns is a language specific fact about English.

Hence: It should be possible for a language to have numerical phrases, explicit counting expressions, that do not force $\mathbf{Z}=$ base $(H E A D)$.
Such a language would allow numerical phrases to apply to prototypical mass nouns, counting portions.

Lima 2014, Khrizman, Landman, Lima, Rothstein and Schvarcz 2015:
This is what happens in the Amazon language Yudja:
No lexical mass-count distinction, all nouns can be counted:
(22) Txabïu apeta pe.

Three blood dripped. (apeta: contextually disjoint portions of blood).

## In sum:

Iceberg semantics: compositional analysis of the mass-count distinction in terms disjointnessoverlap and the head principle.
-Rothstein 2011 observed that measure noun phrases pattern with (mess) mass noun phrases. -I proposed a natural analysis for measures and proved that measure interpretations are mess mass.
-I showed that Rothstein's observation follows from the compositional semantics of bases: The derived interpretations for measure noun phrases are mess mass.
-Distributive interpretations and cardinal-comparison are traditionally standard diagnostics for count nouns.
-The more recent literature showed (surprisingly) that neat mass nouns allow some distributive interpretations and cardinal comparison, despite the fact that neat mass nouns are (true) mass nouns.
-I showed for Dutch (even more surprisingly) that also mess mass nouns allow, in context, distributive interpretations and cardinal comparison.
-I argued that Iceberg semantics gives a natural account for this:
distributive readings and cardinal comparison require linking to a distribution set that is presupposed to be disjoint.
It is only the further assumption that this set be the base of the head that restricts distribution and comparison to count nouns.

If the construction allows linking to a different disjoint set, distribution and cardinal comparison become available for mass nouns.

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